

# SUPERSYMMETRY FOR FLAVORS

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## ABSTRACT

An understanding of the lepton masses within the framework of SUSY is presented. A family symmetry is introduced. Sneutrino VEV breaks this symmetry. The tau mass is due to the Higgs VEV, and muon mass purely from the sneutrino VEV. A viable model is constructed, which predicts  $(1 - 10)$  MeV  $\nu_\tau$ .

## 1. Introduction

The naturalness of the standard model (SM) <sup>1</sup> may require the existence of low energy supersymmetry (SUSY) <sup>2</sup>. The flavor puzzle, namely the fermion masses, mixings and CP violation, of the SM needs new physics to be understood. It would be nice if SUSY also provides (partial) understanding to the flavor problem <sup>3</sup>.

Let us look at the fermion mass pattern. The fact is that the third generation is much heavier than the second generation which is also much heavier than the first. Does this imply a family symmetry? We assume that the answer is yes. Let us consider the charged leptons. By assuming a  $Z_3$  cyclic symmetry among the SU(2) doublets  $L_i$  ( $i = 1, 2, 3$ ) of the three generations <sup>3</sup>, the Yukawa interactions result in a democratic mass matrix. This mass matrix is of rank 1. Therefore only the tau lepton gets mass, the muon and electron are still massless.

The essential point is how the family symmetry breaks. Naively the symmetry breaking can be achieved by introducing more Higgs fields. We consider this problem within SUSY. We have observed that SUSY naturally provides Higgs like fields, which are the scalar neutrinos. Furthermore, if the vacuum expectation values (VEVs) of the sneutrinos are non-vanishing,  $v_i \neq 0$ , the R-parity violating interactions  $L_i L_j E_k^c$ , with  $E_k^c$  denoting the anti-particle superfields of the SU(2) singlet leptons, contribute to the fermion masses, in addition to the Yukawa interactions. This made us to propose that the family symmetry is broken by the sneutrino VEVs <sup>3</sup>.

A remark should be made here. The sneutrino VEV is not enough to the electroweak symmetry breaking (EWSB), the Higgs fields are still necessary. As will be seen,  $v_i$  is typically  $(5 - 10)$  GeV. Such a VEV however, is too large to accommodate the neutrino oscillation data. The point we made is that such a sneutrino VEV breaks a family symmetry which can be useful for the understanding of the charged lepton masses.

Let us focus on the lepton sector. The  $Z_3$  family symmetry results in that Yukawa interactions only give  $\tau$  lepton mass,  $m_\tau \simeq y v_d$ , where  $y$  is the coupling constant, and the Higgs VEV  $v_d \sim v_u \sim 100$  GeV. Taking  $y \sim 10^{-2}$  gives realistic  $m_\tau$ . The muon mass is due to the family symmetry breaking  $v_i \neq 0$ . To make the model phenomenologically acceptable, we found  $v_3 \sim 10$  GeV, and the trilinear R-parity violating couplings  $\lambda \sim 10^{-2}$ . The muon mass is then  $m_\mu \simeq \lambda v_3 \simeq 100$  MeV.

## 2. Challenging Problems

Immediately the following questions are raised to challenge the above described scenario:

1. Is 10 GeV  $v_3$  safe? Namely do we have unacceptable Majoron?
2. Is  $m_{\nu_\tau}$  too large? Roughly it is expected to be  $g_2^2 v_3^2 / M_Z \sim 100 \text{ MeV} - 1 \text{ GeV}$ , with  $g_2$  being the coupling of the SU(2) interaction.
3. Is  $\lambda \sim 10^{-2}$  contradict with the experimental data on the rare decays  $\mu \rightarrow 3e$  and  $\mu \rightarrow e\gamma$ ?

To get rid of the above difficulties, the following things are assumed respectively. First, a large (weak scale) mixing mass term of the Higgs scalar and slepton  $\tilde{L}_3$ ,  $B\mu_3\tilde{H}_u\tilde{L}_3$  is introduced. This term breaks SUSY softly. Because it breaks the lepton number explicitly, no massless Majoron which would be the Goldstone particle corresponding to the spontaneous lepton number violation, appears. Note that the trilinear R-parity violating interactions themselves are not enough to keep the would be Majoron from being light. Second, in the superpotential, the bilinear mixing mass terms, like  $\mu_3 H_u \tilde{L}_3$  and  $\mu H_u H_d$  should be small. We take  $\mu_3 = 0$  and  $\mu = 0$ . In this way,  $m_{\nu_\tau} = 0$  at tree level. This may avoid the difficulty of Question 2. Note that the EWSB is achieved by an alternative superpotential. Third, the family symmetry  $Z_3$  in the ordinary Yukawa and trilinear R-parity violating interactions is adopted. Because of this,  $\mu \rightarrow 3e$  and  $\mu \rightarrow e\gamma$  do not occur. This symmetry can be regarded as accidental at low energy.

The relevant superpotential is

$$\begin{aligned} \mathcal{W} = & y_j \left( \sum_i^3 L_i \right) H_d E_j^c + \lambda_j (L_1 L_2 + L_2 L_3 + L_3 L_1) E_j^c \\ & + \lambda' X (H_u H_d - \mu^2) , \end{aligned} \quad (1)$$

where the last term is for the EWSB with  $\lambda'$  being the coupling constant,  $X$  a singlet superfield and  $\mu$  the weak scale. The baryon number conservation is assumed <sup>4</sup>. Eq. (1) reproduces the lepton masses as we have planned. It would be easier to work in the mass eigenstates of Yukawa interactions,

$$\mathcal{W} = y^\tau L_\tau H_d E_\tau^c + L_e L_\mu (\lambda_\mu E_\mu^c + \lambda_\tau E_\tau^c) + \lambda' X (H_u H_d - \mu^2) , \quad (2)$$

where the left-handed leptons are

$$\begin{aligned} \tau_L &= \frac{1}{\sqrt{3}}(e_1 + e_2 + e'_3) , \\ \mu_L &= \frac{1}{\sqrt{2}}(e_1 - e_2) , \\ e_L &= \frac{1}{\sqrt{6}}(e_1 + e_2 - 2e'_3) , \end{aligned} \quad (3)$$

with  $(\nu_i, e_i)$  being the fermionic component of  $L_i$ .  $(\nu'_i, e'_i)$  means the physical leptons after considering the mixing with the neutralinos and charginos. From Eq. (2), we see that  $\mu \rightarrow 3e$  does not occur. Instead, the rare decays  $\tau \rightarrow 2e\mu$  and  $3\mu$  have branching ratios  $10^{-7}$  if  $m_{\tilde{\nu}_i} \simeq 100 \text{ GeV}$ .

At the quantum level, a comparatively large neutrino mass is inevitably induced due to the large lepton number violating effect in  $B\mu_3$  mass. It occurs at the one-loop level with a Zino exchange. Therefore  $m_{\nu_\tau} \neq 0$  at one loop which will be studied further. Is it natural in a theory in which  $B\mu_3$  is large,  $\mu_3$  is vanishingly small and  $m_{\nu_\tau}$  is consistent with experiment?

### 3. A Model of GMSB

With the framework of gauge mediated SUSY breaking (GMSB) <sup>5</sup>, the scenario asked in the last question can be realized naturally <sup>6</sup>. Lepton number violation is introduced originally in the messenger sector. It is then communicated to the SM sector including the related soft SUSY breaking terms. We will make use of the observation of the  $\mu$ -problem in GMSB <sup>7</sup>. It was noted that both  $\mu$  term and its corresponding soft breaking  $B\mu$  term can be generated at one loop. Either  $\mu$  is at the weak scale and  $B\mu$  is unnaturally large, or  $B\mu$  is at the weak scale and  $\mu$  is very small. This is not a problem in our model, because the EWSB given in Eq. (1) does not need the  $\mu$  term. However, we apply similar observation to the discussion of the mixing of  $H_u$  and  $L_3$ .

The messengers are introduced as follows with the  $SU(3) \times SU(2) \times U(1)$  quantum numbers,

$$S, S' = (1, 2, -1) , \quad \bar{S}, \bar{S}' = (1, 2, 1) , \quad (4)$$

and

$$T, T' = (3, 1, -2/3) , \quad \bar{T}, \bar{T}' = (\bar{3}, 1, 2/3) . \quad (5)$$

Two more gauge singlets are introduced  $Y$  for the SUSY breaking and  $V$  for the lepton number violation. The superpotential is then

$$\mathcal{W}_{\text{total}} = \mathcal{W} + \mathcal{W}_1 + \mathcal{W}_2 , \quad (6)$$

where

$$\begin{aligned} \mathcal{W}_1 = & m_1(\bar{S}'S + S'\bar{S}) + m_2(\bar{T}'T + T'\bar{T}) + m_3S\bar{S} + m_4T\bar{T} + m_5V^2 \\ & + Y(\lambda_1S\bar{S} + \lambda_2T\bar{T} + \lambda_3V^2 - \mu_1^2) , \end{aligned} \quad (7)$$

$$\mathcal{W}_2 = V(\lambda_5H_uS + \lambda_6L_3\bar{S}) , \quad (8)$$

with  $\mu_1$  being the SUSY breaking scale. The lepton number violation lies in  $\mathcal{W}_2$ . By integrating out the heavy messengers, the effective Lagrangian related to lepton number violation is

$$\mathcal{L}_{\text{eff}}^{\psi} = \mu_3L_3H_u|_{\theta\theta} + B\mu_3\tilde{L}_3\tilde{H}_u + \text{h.c.} , \quad (9)$$

where

$$\mu_3 \simeq \frac{\lambda_5\lambda_6}{16\pi^2} \frac{\mu_1^2}{m_3} , \quad B\mu_3 \simeq \frac{\lambda_5\lambda_6}{16\pi^2} \left( \frac{\mu_1^2}{m_3} \right)^2 . \quad (10)$$

Or

$$B\mu_3 = \mu_3 \frac{\mu_1^2}{m_3} . \quad (11)$$

This is what we needed.  $\mu_1^2/m_3$  is usually around 100 TeV.  $\mu_3$  is very small if  $B\mu_3$  is taken to be the weak scale.

From the scalar potential, the sneutrino VEV is obtained as

$$v_1 = 0, \quad v_2 = 0, \quad v_3 = -\frac{B\mu_3 v_u}{M_A^2 + \frac{1}{2}M_Z^2 \cos 2\beta}. \quad (12)$$

Numerically,  $v_3 \sim 10$  GeV if  $M_A \sim 300$  GeV and  $B\mu_3 \sim (60 \text{ GeV})^2$ . Nonzero  $v_3$  implies the mixing between neutrino  $\nu_3$  and the neutralinos. In addition,  $B\mu_3$  causes comparatively large  $\nu_3$ -Higgsino mixing at one loop,

$$m_{3H} \simeq \mu_3 + \frac{g_2^2}{16\pi^2} \frac{B\mu_3}{M_{\tilde{Z}}} \sim 0.04 - 0.1 \text{ GeV}. \quad (13)$$

The  $\tau$  neutrino mass should be obtained from the full neutralino mass matrix,

$$-i(\nu_3 \quad H_d^0 \quad H_u^0 \quad \tilde{Z} \quad X) \begin{pmatrix} 0 & 0 & m_{3H} & av_3 & 0 \\ 0 & 0 & 0 & av_d & \lambda'v_u \\ m_{3H} & 0 & 0 & -av_u & \lambda'v_d \\ av_3 & av_d & -av_u & M_{\tilde{Z}} & 0 \\ 0 & \lambda'v_u & \lambda'v_d & 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_3 \\ H_d^0 \\ H_u^0 \\ \tilde{Z} \\ X \end{pmatrix} + \text{h.c.}, \quad (14)$$

with  $a = (\frac{g_1^2 + g_2^2}{2})^{1/2}$ . It gives

$$m_{\nu_\tau} \simeq \frac{m_{3H}v_3}{M_Z} \sim (1 - 10) \text{ MeV}. \quad (15)$$

This heavy  $\nu_\tau$  can decay to  $e^+e^-\nu_e$ . We later noted that  $\nu_\tau$  can decay to gravitino + photon with a longer lifetime. In writing down the expression of the physical  $\nu_\tau$  and  $\tau$  states in Refs. <sup>3</sup> and <sup>6</sup>, the gaugino masses were neglected. In fact,  $\nu_3$  mixes with photino.

#### 4. Discussion

The idea about fermion masses presented in this model essentially depends on SUSY. A model of large sneutrino VEV exists.  $(1 - 10) \text{ MeV}$   $\nu_\tau$  is a consequence of this VEV. It should be noted that  $L_3$  and  $H_d$  appear in the superpotential in different ways, so that the VEV cannot be rotated away through redefining Higgs superfield.

The atmospheric neutrino anomaly must be explained by introducing a sterile neutrino which is also necessary from the constraint of the Big-Bang Neucleosynthesis. The extension of this idea to the quark sector can be found in Ref. <sup>8</sup>.

The  $B\mu_3$  term breaks the family symmetry explicitly. It would be more appealing if the family symmetry breaking is spontaneous in some clever model.

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